Further Mathematic preparation work

The tasks below need to completed and submitted in your first lesson. The questions covered are based on the higher GCSE content and the marks stated will help you to see the amount of working expected. When completing these questions, think carefully about your mathematical presentation as this will be vital in helping you to be successful at A-level.

**Task 1:**

This should take you approximately 1 hour to complete based on the marks available.

1. Simplify these expressions as far as possible.
   
   \[ a \frac{x^2 - 2x - 3}{x^2 + 2x + 1} \]  
   \[ b \frac{x^2 - 25}{x^2 + 6x + 8} + \frac{x^2 - 2x - 15}{x^2 - 16} \]  
   (3 marks)  
   (4 marks)

2. The line \( l \) is a tangent to the circle \( x^2 + y^2 = 20 \) at the point \( P(2, 4) \).
   
   The tangent intersects the \( y \)-axis at point \( A \). Find the area of the triangle \( OPA \).  
   (5 marks)

3. Expand and simplify \((\sqrt{p} + 2\sqrt{q})(2\sqrt{p} - \sqrt{q})\)  
   (3 marks)

4. a. Write \( 3x^2 - 12x + 7 \) in the form \( a(x+b)^2 + c \)  
   (3 marks)

   b. Hence, or otherwise, write down the coordinates of the turning point of the graph of \( y = 3x^2 - 12x + 7 \)  
   (1 mark)

5. Prove algebraically that the product of three consecutive odd numbers is always an odd number.  
   (4 marks)

6. The functions \( g \) and \( f \) are defined as \( g(x) = \frac{2x}{4-x} \) and \( f(x) = 3x - 1 \).
   
   Given that \( x \neq 4 \), find the value(s) of \( x \) such that \( g(x) = f(x) \), giving your answer(s) to 2 decimal places.  
   (6 marks)

7. The line \( l_1 \) has equation \( y = -\frac{1}{2}x + 3 \) and intersects the \( x \)- and \( y \)-axes at the points \( A \) and \( B \) respectively.
   
   a. Find the exact length of the line segment \( AB \).  
   (3 marks)

   b. Find the equation of the line \( l_2 \) perpendicular to \( l_1 \) which passes through the point \( P(-1, -2) \).  
   (2 marks)
The line $l_2$ intersects $l_1$ at the point $C$.

c Find the midpoint of the line segment $AC$. (4 marks)

8 A triangle $ABC$ has side lengths $AB = 10$ cm, $BC = 15$ cm and $AC = 8$ cm.

a Find the size of the largest angle, giving your answer to 2 decimal places. (3 marks)

b Find the area of the triangle, giving your answer to 2 decimal places. (2 marks)

9 a Sketch the graph of $y = \cos x$ for $-180 \leq x \leq 360^\circ$, showing the points where the graph cuts the axes. (2 marks)

b Hence find the exact values of $x$ in the interval $-180 \leq x \leq 360^\circ$ for which $\cos x = -\frac{3}{2}$ (3 marks)

**Task 2:**

Complete the questions below. Showing your working is important. Question 4 is there for us to see how you approach unfamiliar questions so do not worry if you are unable to complete it.

1. Simplify

   a. $(x + h)^2 + (x - h)^2$

   b. $(2\sqrt{t} - 3)(1 + \sqrt{t})$

   c. $\frac{3t^2}{(1+t)^2} ÷ \frac{t^2}{(1+t)^3}$

   d. $\frac{\sqrt{x}}{\sqrt{x-1}} \times \frac{\sqrt{x}}{\sqrt{x+1}}$

   e. $\frac{1 + \frac{1}{y}}{1 - \frac{1}{xy}}$

   f. $\sqrt{\frac{1-\frac{2t}{1+t^2}}{1+\frac{2t}{1+t^2}}}$

2. Factorise

   a. $x(x + 1)^2 + (x + 1)(x^2 - 3)$

   b. $5(x + 1)^2 + 7x(x + 1)$

   c. $(x - 2)^3 + 5x(x - 2)^2$

3. Make the letter stated the subject

   a. $b = a(1 - c), \quad c$

   b. $\frac{y-k}{a} = \frac{x-h}{b}, \quad y$
c. \( T = 2\pi \sqrt{\frac{l}{g}} \), \( g \)

d. \( 3mc = (4 + 3m)(c - 4) \), \( c \)

e. \( m = \frac{x+a}{b-x} \), \( x \)

f. \( 2x - 3y - 3mx + 2my - 2m + 4 = 0 \), \( m \)

4. The first five terms of an arithmetic sequence are:

\[ x + 1, 2x, \frac{2(2x + 3)}{6 - x}, x^2 - 2, 5x - 3 \]

Show that the term \( 4x^2 - 3 \) is not in the sequence.